INTERSECTION OF TWO PLANE SHOCK WAVES IN SPACE

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We consider the problem of the intersection of two plane shock waves in space. We show that, unlike the two-dimensional case which has a unique solution, in space there exists a region of variation of parameters defining the problem, in which two solutions with supersonic velocity behind the corresponding systems of shock waves are possible.

Let us consider the intersection of two plane shock waves. The condition of equilibrium of the flow behind them requires that, generally speaking, shock waves should be refracted on passing through the plane of intersection, and the angle between them should, with one exception given in [1], also change.

The symmetrical case when the initial direction of the flow is parallel to the plane bisecting the angle between the discontinuities, is equivalent to the reflection of the shock wave from a wall.

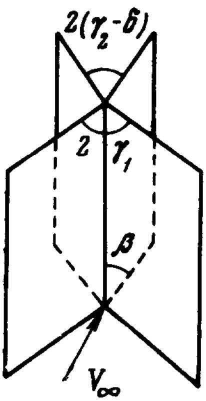

FIG. 1*a*

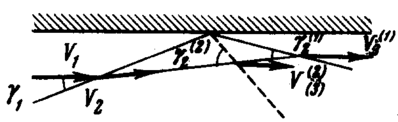

FIG. 1*b*

In the present work we shall limit ourselves to the symmetrical case. The non-symmetrical case is basically similar to the symmetrical one, and its qualitative solution should produce analogous results.

By analogy with the case of reflection from the wall, we shall call the shock wave in front of the intersection 'incident', and behind the intersection 'reflected'. We shall assume that both, incident and reflected waves, are weak. Here 'weak' means an oblique shock wave, the velocity on both sides of which, is supersonic. We know, that out of two solutions of the system of equations for an oblique shock wave, is the weak shock wave that occurs on the wedge.

If we take the value of M as finite, then the condition of the flow being supersonic behind the shock wave narrows the region in which weak shock waves exist somewhat, but the difference when compared with the value usually accepted in the classical gas dynamics [1], is small. The corresponding difference in the maximum angle of the wedge, does not exceed 0.5°.

The parameters defining the problem of intersection of two plane shock waves in space are: the angle $2\gamma_1$ between their planes, the number M_∞ of the undisturbed flow and the angle β of the inclination of its velocity vector to the line of intersection of the shock waves (fig. 1).

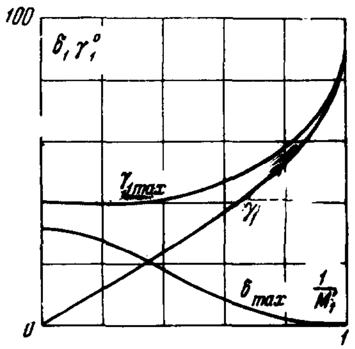

FIG. 2

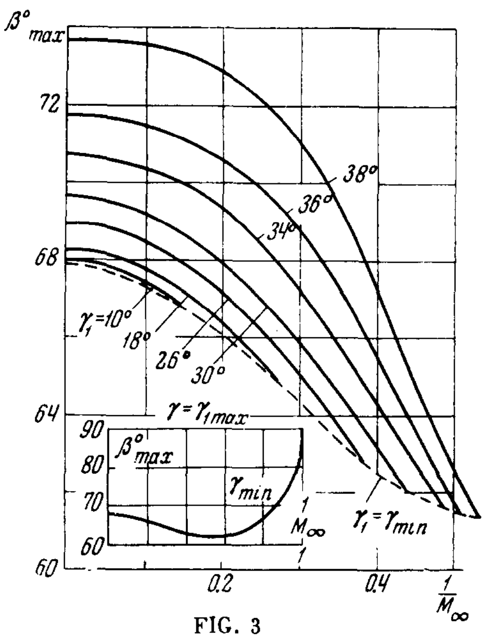

FIG. 3

The region of variation of parameters M_∞, β and γ is limited by the condition of existence of the incident and reflected shock waves

$$\frac{1}{M_\infty \sin\beta} \leqslant \sin\gamma_1 \leqslant \sin\gamma_{1\max} \tag{1}$$

The value of $\gamma_{1\ max}$ is determined later from the condition of regularity of reflection of the shock waves.

With the above parameters, the angle between the planes of the shock waves and the velocity vector of the undisturbed flow is equal to

$$\omega_1 = \sin^{-1}(\sin\beta\ \sin\gamma_1) \tag{2}$$

Obviously, within the considered flow, the component of velocity projected on the line of intersection of shock waves, has a constant value equal to $V_\infty \cos\beta$. Consequently, the parameters of the flow can be computed over the plane perpendicular to this line and moving down it with the velocity equal to $V_\infty \cos\beta$. We shall consider the problem in this plane with the view determining true values of the density, pressure and temperature in the surrounding space.

Slip velocity should be accounted for in the calculations of the direction and magnitude of velocity.

The component of the flow velocity in the plane perpendicular to the line of intersection of the shock waves, corresponds to the number $M_1' = M_\infty \sin\beta$, while the angle it forms with the direction of the incident wave is equal to γ_1 (fig. 1*b*).

Direction of the component of the velocity behind the reflected wave V_3 coincides, in

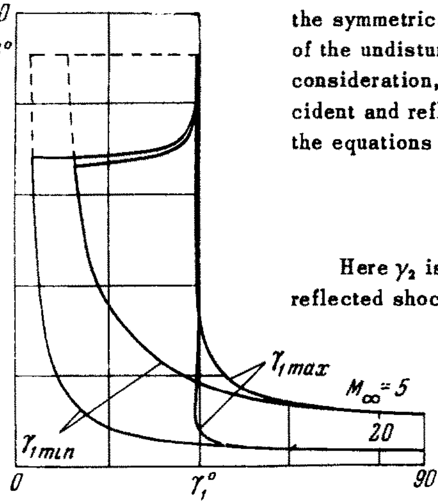

Fig. 4

the symmetrical case, with the direction of the velocity componen of the undisturbed flow $V_1 = V_\infty \cos\beta$, hence in the plane under consideration, the angle of rotation δ of velocity across the incident and reflected wave, has the same numerical value. From the equations for an oblique shock wave, if follows [1]

$$\frac{V_3}{V_1} = \frac{\cos\gamma_1}{\cos(\gamma_1-\delta)}\,\frac{\cos\gamma_2}{\cos(\gamma_2-\delta)} \tag{3}$$

Here γ_2 is the angle between the incident velocity and the reflected shock wave

$$\tan(\gamma_i-\delta) = \frac{\varkappa-1}{\varkappa+2}\tan\gamma_i + \frac{2}{\varkappa+1}\,\frac{1+\tan^2\gamma_1}{M_i'^2\tan\gamma_i} \tag{4}$$

$$M'_{i+1} = \frac{1}{\sin(\gamma_i-\delta)}\left(\frac{2(\varkappa-1)^{-1}+M_i'^2\sin^2\gamma_i}{2\varkappa(\varkappa-1)^{-1}M_i'^2\sin^2\gamma_i-1}\right. \tag{5}$$

$$p_1^\circ = \frac{p_2/p_\infty-1}{{}^1\!/_2\varkappa M_\infty^2} = \frac{4\sin^2\beta}{\varkappa+1}\left(\sin^2\gamma_1-\frac{1}{M_1'^2}\right) \tag{6}$$

$$p_2^\circ = \frac{p_3/p_\infty-1}{{}^1\!/_2\varkappa M_\infty^2} = \frac{4\sin^2\beta}{\varkappa+1}\left(\sin^2\gamma_2-\frac{1}{M_2'^2}\right)\frac{\sin 2\gamma_1}{\sin 2(\gamma_1-\delta)} + p_1^\circ \tag{7}$$

$$\frac{\rho_3}{\rho_\infty} = \frac{\tan\gamma_1}{\tan(\gamma_1-\delta)}\,\frac{\tan\gamma_2}{\tan(\gamma_2-\delta)}$$

$$\frac{T_3}{T_\infty} = \frac{p_3}{p_\infty}\,\frac{\rho_\infty}{\rho_2} \tag{8}$$

Here $\varkappa$ is the ration of specific heats, indices $i = 1$ and 2 denote the parameters of the incident and reflected shock wave, 2 and 3 denote the parameters of the respective flows behind them and ∞ denotes the parameters of the undisturbed flow.

The condition of regularity of the intersection of two plane shock waves in space when it gives rise to two shock waves is, that in the plane considered, the angle δ should not exceed the maximum angle of the wedge possessing an attached shock wave in the flow whose velocity behind the shock wave is M_2'.

This magnitude δ_{max} and the corresponding value of $\gamma_{1\,max}$, can be found by numerica methods from the system (4) and (5) for $i = 1$ and 2, and they depend only on $M_1' = M_\infty \sin$ (see fig. 2 where $\gamma_1 = \gamma_{1min}$).

In space, angles of rotation of velocity across the incident and reflected waves are different from each other, and the final direction of velocity behind the considered system of shock waves (with the slip velocity taken into account) does not coincide with the initial direction of the undisturbed flow. Both vectors however, are parallel to the plane of symmetry, and the angle τ between them is equal to

$$\tau = \beta - \theta, \qquad \theta = \tan^{-1}\left(\frac{V_3}{V_1}\tan\beta\right) \tag{9}$$

where θ is the angle between the downstream velocity vector and the line of intersection of the shock waves, and the value of M corresponding to the above velocity, is

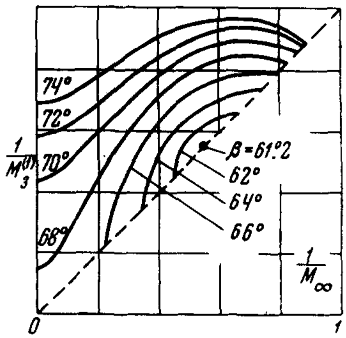

FIG. 5

$$M_3 = \frac{M_3'}{\sin\theta} \qquad (10)$$

The angle between the planes of reflected shock waves is equal to $2(\gamma_2 - \delta)$, while their inclination to the velocity vector immediately in front of them, is

$$\omega_2 = \cos^{-1}\left(\frac{1+\tan^2\theta\cos^2(\gamma_2-\delta)}{1+\tan^2\theta\cos^2(\gamma_2-\delta)\cos^2\gamma_2}\right)^{1/2} \qquad (11)$$

In the plane case of intersection of two shock waves ($\beta = \frac{1}{2}\pi$) only one of the solutions of (4) corresponds to the weak reflected wave with supersonic velocity behind it, consequently the solution of the problem is unique.

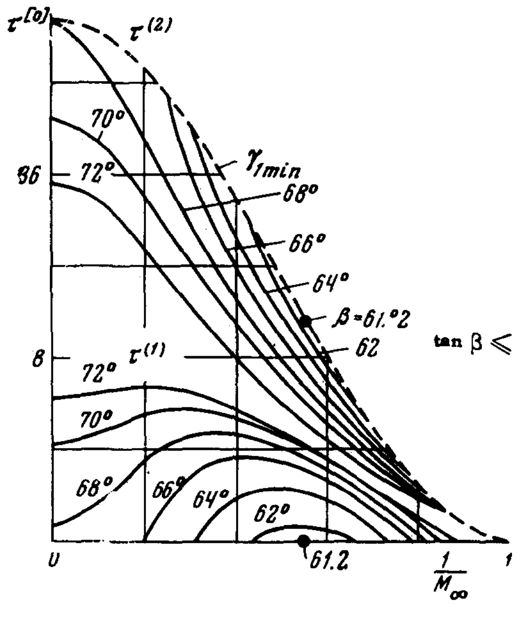

FIG. 6

In the three-dimensional space, both solutions of (4), $\gamma_1^{(1)}$ and $\gamma_2^{(2)}$ for the reflected wave in the auxiliary plane correspond to weak shock waves, if $M_3' > \sin\theta$. Assuming

$$\sin\theta_{max} = M_3'^{(2)} \qquad (12)$$

we obtain, from (9)

$$\tan\beta \leqslant \frac{\cos(\gamma_1-\delta)}{\cos\gamma_1}\,\frac{\cos(\gamma_2^{(2)}-\delta)}{\cos\gamma_2^{(2)}}\tan\theta_{max} \qquad (13)$$

Figure 3 gives the values of β_{max} for $M_\infty \geqslant 2$. When $\gamma_1 = \gamma_{1min}$, and $\delta = 0$, we have $\gamma_2^{(2)} = \frac{1}{2}\pi$, and the considered system of shock waves degenerates, in the second solution, into a single planar shock wave inclined to the velocity of the undisturbed flow at the angle β. When $\gamma_1 = \gamma_{1\,max}$, both solutions coincide, $\gamma_2^{(2)} = \gamma_2^{(1)}$ $\beta = \theta = \frac{1}{2}\pi$.

The inequalities (1) and (13) limit the region of variation of defining parameters, in which the problem of regular intersection of two shock waves in space, has two solutions with weak reflected shock waves, behind both of which the velocity is supersonic.

In one of them, the projection of this velocity on the plane perpendicular to the line of intersection of the shock waves is also supersonic (first solution $M_3'^{(1)} > 1$), while in the other the projection is subsonic (second solution — $M_3'^{(2)} \leqslant 1$).

Solid lines on fig. 4 show these regions for $M_\infty = 20$ and 5. The region situated to the right of the curve $\gamma_{1\,max}$ corresponds to the cases with the Mach reflection of the wave. Broken lines map the region in which only one solution with a weak reflected shock wave exists, and for which the projection of the resultant velocity onto the plane perpendicular to the line of intersection of the shock waves, is supersonic.

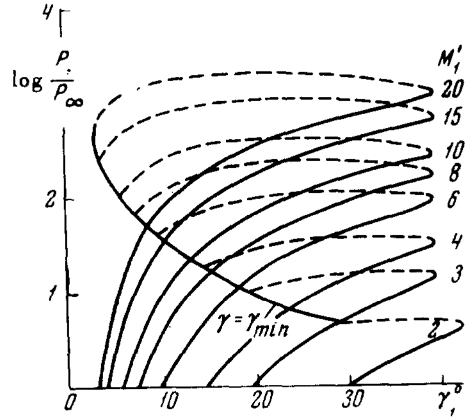

FIG. 7

Second solution in this region corresponds to a strong reflected shock wave, behind which the velocity is subsonic. The line separating these two regions corresponds to the case when the velocity behind the reflected shock waves is, in the second solution, equal to the speed of sound $M_3^{(2)} = 1$.

Corresponding values of $M_3^{(1)}$ for the first solution are given on fig. 5, while fig. 6 shows, for both solutions, the angles of inclination $\tau^{(i)}$ of the velocity vector behind the shock waves, to the velocity of the undisturbed flow. Pressure, density and temperature depend, within the region of the considered flow, together with $\tan\beta/\tan\theta$, on the combination of the defining parameters $M_1' = M_\infty \sin\beta$ and γ_1. Figs. 7 and 8 give the density and pressure in the region behind the reflected waves for the first and second solution, solid lines referring to the first solution $\gamma_2^{(1)}$ broken lines to the second one $\gamma_2^{(2)}$. It is clear that in the second solution (in which the projection of the velocity component on the plane perpendicular to the line of intersection of the shock waves is subsonic), variation of the parameters of the flow in the system of shock waves is much larger, then in the first solution.

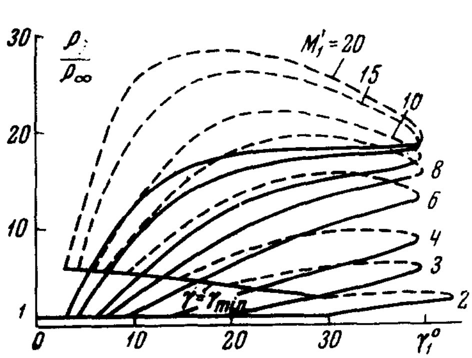

FIG. 8

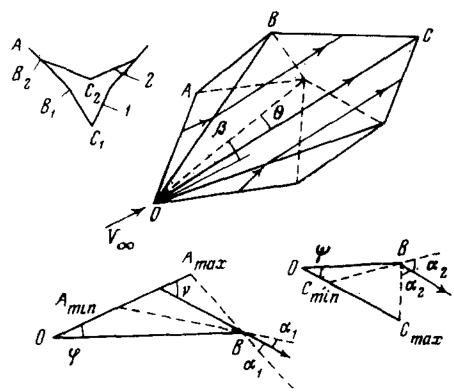

FIG. 9

This, or some other solution, should exist for the flow around the surface formed by the velocity vectors of the field of the corresponding flow. Fig. 9 shows an example of such a body consisting of four plane forces, edges of which support the shock waves shown on the figure with broken lines.

Streamlines on the faces are indicated by means of arrows. The edge OC coincides with the direction of the velocity behind the reflected waves. Various surfaces with different positions of the faces OBC, correspond to various solutions. Their transverse cross-sections are shown on fig. 9.

For some shock wave configurations, the surface AB_2C_2 degenerates into a V-shaped wing, in vicinity of the edge OC of which, a region of excess pressure situated behind the reflected wave, exists.

A necessary condition for evolving a numerical computation scheme for the flow in proximity of the body is, that the perturbations originating on its trailing edge should not intersect any faces and, that the pressure on these faces should be constant. Considerations of the regions of perturbation originating from the trailing edges of the faces OAB and OBC (see broken lines on fig. 9) lead us to conclusion that the condition for the existence of a flow solvable by numerical methods, is

$$0 \leqslant \frac{OC}{OA} \leqslant \frac{\sin(\alpha_1 + \nu)\sin(\alpha_2 + \psi)}{\sin(\alpha_1 + \nu - \varphi)\sin\alpha_2} \qquad (14)$$

$$\nu + \alpha_1 < \pi, \qquad \psi + \alpha_2 < \pi$$

Here φ and $\psi = f_i(\varphi, \gamma_1, \beta, M_\infty)$ are the face angles at the apex of the body, α_1 and α_2 are the Mach angles in the region behind the incident and reflected shock waves (with the slip belocity taken into account), $\nu = f(\varphi, \gamma_1, \beta, M_\infty)$ is the inclination of the velocity vector on the face OAB to the front edge OA. For regulat intersection of the shock waves, we always have $\nu > \alpha_1$.

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